Proton transversity and intrinsic motion of the quarks *

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The spin structure of the system of quasifree fermions having total angular momentum $J = 1/2$ is studied in a consistently covariant approach. Within this model the relations between the spin functions are obtained. Their particular cases are the sum rules Wanzura - Wilczek, Efremov - Leader - Teryaev, Burkhardt - Cottingham and also the expression for the Wanzura - Wilczek twist 2 term g_2^{WW}. With the use of the proton valence quark distributions as an input, the corresponding spin functions including transversity are obtained.

I. INTRODUCTION

In this talk some results following from the covariant quark-parton model (QPM) will be shortly discussed, details of the model can be found in our recent papers [1] [2] [3]. In this version of QPM valence quarks are considered as quasifree fermions on mass shell. Momenta distributions describing the quark intrinsic motion have spherical symmetry corresponding to the constraint $J = 1/2$, which represents the total angular momentum - generally consisting of spin and orbital parts. I shall mention the following items:

1. What sum rules follow from this approach for the spin structure functions g_1 and g_2?

2. How can these structure functions be obtained from the valence quark distributions u_V and d_V - if the $SU(6)$ symmetry is assumed?

3. Why the first moment Γ_1 calculated in this approach can be substantially less, than the corresponding moment calculated within the standard, non covariant QPM, which is based on the infinite momentum frame?

4. Calculation of the transversity distribution.

II. MODEL

The model is based on the set of distribution functions $G_{k,\lambda}(\frac{pP}{M})$, which measure probability to find a quark in the state:

$$u(p, \lambda \mathbf{n}) = \frac{1}{\sqrt{N}} \begin{pmatrix} \phi_{\lambda \mathbf{n}} \\ \frac{\mathbf{p}\sigma}{p_0+m} \phi_{\lambda \mathbf{n}} \end{pmatrix}; \qquad \frac{1}{2}\mathbf{n}\sigma \phi_{\lambda \mathbf{n}} = \lambda \phi_{\lambda \mathbf{n}}, \qquad \lambda = \pm\frac{1}{2},$$

where $\mathbf{n}$ coincides with the direction of the proton polarization $\mathbf{J}$. Correspondingly, m and p are quark mass and momentum, similarly M and P for the proton. With the use of these distribution functions one can define the function H, which in the target rest frame reads:

$$H(p_0) = \sum_{k=1}^{3} e_k^2 \Delta G_k(p_0); \qquad \Delta G_k(p_0) = G_{k,+1/2}(p_0) - G_{k,-1/2}(p_0), \tag{1}$$

In the previous study it was shown [1], how the spin structure functions can be obtained from the generic function H. If one assume $Q^2 \gg 4M^2x^2$, then:

$$g_1(x) = \frac{1}{2} \int H(p_0) \left(m + p_1 + \frac{p_1^2}{p_0+m} \right) \delta\left(\frac{p_0+p_1}{M} - x \right) \frac{d^3p}{p_0}; \quad x = \frac{Q^2}{2M\nu},$$

$$g_2(x) = -\frac{1}{2} \int H(p_0) \left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0+m} \right) \delta\left(\frac{p_0+p_1}{M} - x \right) \frac{d^3p}{p_0}.$$

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Let me remark, that procedure for obtaining the functions g_1, g_2 from the distribution H is rather complex, nevertheless the task is well-defined and unambiguous. For the transversity the corresponding expression reads

$$\delta q(x) = \varkappa \int H\left(p_0\right)\left(Mx - \frac{p_T^2/2}{p_0 + m}\right)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{d^3p}{p_0},$$

where the factor $\varkappa$ depends on the approach applied in the calculation [3].

III. SUM RULES

One can observe, that the functions above have the same general form

$$\int H(p_0)f(p_0, p_1, p_T)\delta\left(\frac{p_0 + p_1}{M} - x\right)d^3p \tag{2}$$

and differ only in kinematic term f. This integral, due to spheric symmetry and presence of the $\delta-$function term, can be expressed as a combination of the momenta:

$$V_n(x) = \int H(p_0)\left(\frac{p_0}{M}\right)^n\delta\left(\frac{p_0 + p_1}{M} - x\right)d^3p. \tag{3}$$

One can prove [2], that these functions satisfy

$$\frac{V_j'(x)}{V_k'(x)} = \left(\frac{x}{2} + \frac{x_0^2}{2x}\right)^{j-k}; \qquad x_0 = \frac{m}{M}.$$

This relation then gives possibility to obtain integral relations between different functions having form (3) or (2), in particular for $g_1(x)$ and $g_2(x)$ one gets for $m \to 0$:

$$g_2(x) = -g_1(x) + \int_x^1 \frac{g_1(y)}{y}dy,$$

$$g_1(x) = -g_2(x) - \frac{1}{x}\int_x^1 g_2(y)dy.$$

Another rule, which is obtained in this approach, reads:

$$\int_0^1 x^\alpha\left[\frac{\alpha}{\alpha + 1}g_1(x) + g_2(x)\right]dx = 0,$$

which is valid for *any* α, for which the integral exists. For $\alpha = 2, 4, 6, ...$ the relation corresponds to the Wanzura - Wilczek sum rules [4]. Other special cases correspond to the Burkhardt - Cottingham [5] ($\alpha = 0$) and the Efremov - Leader - Teryaev [6] (ELT, $\alpha = 1$) sum rules. For the transversity one gets

$$\delta q(x) = 2\varkappa\left(g_1(x) + \int_x^1 \frac{g_1(y)}{y}dy\right).$$

IV. VALENCE QUARKS

Now I shall apply the suggested approach to the description of the real proton. For simplicity I assume:

1) Spin contribution from the sea of quark-antiquark pairs and gluons can be neglected, so the proton spin is generated only by the valence quarks. The negligible contribution from the quark sea was recently reported in [7].

2) In accordance with the non-relativistic *SU(6)* approach, the spin contribution of individual valence terms is given by the fractions:

$$s_u = 4/3, \qquad s_d = -1/3.$$

If the symbols h_u and h_d denote momenta distributions of the valence quarks in the proton rest frame, which are normalized as

$$\frac{1}{2} \int h_u(p_0) d^3 p = \int h_d(p_0) d^3 p = 1,$$

then the generic distribution (1) reads

$$H(p_0) = \sum e_j^2 \Delta h_j(p_0) = \left(\frac{2}{3}\right)^2 \frac{2}{3} h_u(p_0) - \left(\frac{1}{3}\right)^2 \frac{1}{3} h_d(p_0). \tag{4}$$

In the paper [10], using a similar approach, I studied also the unpolarized structure functions. Structure function F_2 can be expressed as

$$F_2(x) = x^2 \int G(p_0) \frac{M}{p_0} \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3 p; \qquad G(p_0) = \sum_q e_q^2 h_q(p_0).$$

On the other hand, for proton valence quarks one can write

$$F_2(x) = \frac{4}{9} x u_V(x) + \frac{1}{9} x d_V(x),$$

so combination of the last two relations gives:

$$q_V(x) = x \int h_q(p_0) \frac{M}{p_0} \delta\left(\frac{p_0 + p_1}{M} - x\right) d^3 p; \qquad q = u, d.$$

Since this is again the integral having the structure (2), one can apply the technique of integral transforms and (instead of relations between $g_1, g_2, \delta q$) obtain the relations between $g_j^q, \delta q$ and q_V. For $m \to 0$ these relations read:

$$g_1^q(x) = \frac{\cos \eta_q}{2} \left(q_V(x) - 2x^2 \int_x^1 \frac{q_V(y)}{y^3} dy \right),$$

$$g_2^q(x) = \frac{\cos \eta_q}{2} \left(-q_V(x) + 3x^2 \int_x^1 \frac{q_V(y)}{y^3} dy \right),$$

$$\delta q(x) = \varkappa \cos \eta_q \left(q_V(x) - x^2 \int_x^1 \frac{q_V(y)}{y^3} dy \right),$$

where $\cos \eta_q$ is corresponding $SU(6)$ factor. Now, taking quark charges and the $SU(6)$ factors as in Eq. (4), one can directly calculate g_1, g_2 and δq only using the input on the valence quark distributions $q_V = u_V, d_V$. More detailed discussion of obtained results is done in our cited papers. Here I mention our (A.Efremov, O.Teryaev and P.Zavada) quite recent result on double spin asymmetry $A_{TT}(y, Q^2)$, which is shown in Fig. (1).
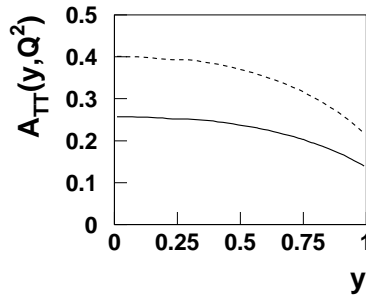

FIG. 1. Double spin asymmetry at $Q^2 = 4 GeV/c$ is calculated using two transversity approaches: Interference effects are attributed to quark level only *(solid line)*. Interference effects at parton-hadron transition stage are included in addition *(dashed line)*, this curve represents upper bound only.

Calculation is done in an accordance with the recent study [8], but with the use of quark trasversities obtained in the presented approach, which gives slightly lower estimation of the A_{TT}. It is expected, that this function could be accessible in the PAX experiment [9], which will measure spin asymmetry of the lepton pairs produced in collisions of transversely polarized protons and antiprotons.

Further, in this talk I want concentrate on the discussion and explanation, why intrinsic quark motion substantially reduces the first moment of the spin function g_1. I have shown [1], that

$$\Gamma_1 \equiv \int g_1(x)dx = \frac{1}{2}\int H(p_0)\left(\frac{1}{3} + \frac{2m}{3p_0}\right)d^3p, \tag{5}$$

which, in the $SU(6)$ approach gives

$$\frac{5}{18} \geq \Gamma_1 \geq \frac{5}{54},$$

where left limit is valid for the static and right one for massless quarks. In other words, it seems:

$$\textit{more intrinsic motion} \Leftrightarrow \textit{less spin}$$

All right, this is a mathematical result, but how to understand it from the point of view of physics?

First, forget structure functions for a while and calculate completely another task. Let me remind general rules concerning angular momentum in quantum mechanics:

1) Angular momentum consist of orbital and spin part: **j=l+s**

2) In the relativistic case **l** and **s** are not conserved separately, only total angular momentum **j** is conserved. So, one can have pure states of $j(j^2, j_z)$ only, which are for fermions with $s = 1/2$ represented by the relativistic spheric waves [11]:

$$\psi_{jlj_z}(\mathbf{p}) = \frac{1}{\sqrt{2p_0}}\left(\begin{array}{c} i^{-l}\sqrt{p_0 + m}\,\Omega_{jlj_z}\left(\frac{\mathbf{p}}{p}\right) \\ i^{-l'}\sqrt{p_0 - m}\,\Omega_{jl'j_z}\left(\frac{\mathbf{p}}{p}\right) \end{array}\right); \qquad j = l \pm \frac{1}{2}, \qquad l' = 2j - l,$$

$$\Omega_{l+1/2,l,j_z}\left(\frac{\mathbf{p}}{p}\right) = \left(\begin{array}{c} \sqrt{\frac{j+j_z}{2j}}Y_{l,j_z-1/2} \\ \sqrt{\frac{j-j_z}{2j}}Y_{l,j_z+1/2} \end{array}\right),$$

$$\Omega_{l'-1/2,l',j_z}\left(\frac{\mathbf{p}}{p}\right) = \left(\begin{array}{c} -\sqrt{\frac{j-j_z+1}{2j+2}}Y_{l',j_z-1/2} \\ \sqrt{\frac{j+j_z+1}{2j+2}}Y_{l',j_z+1/2} \end{array}\right).$$

This wavefunction is simplified for the state with total angular momentum (spin) equal 1/2:

$$j = j_z = \frac{1}{2}, \qquad l = 0 \qquad \Rightarrow \qquad l' = 1,$$

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = i\sqrt{\frac{3}{4\pi}}\cos\theta, \qquad Y_{11} = -i\sqrt{\frac{3}{8\pi}}\sin\theta\exp(i\varphi),$$

which gives

$$\psi_{jlm}(\mathbf{p}) = \frac{1}{\sqrt{8\pi p_0}}\left(\begin{array}{c} \sqrt{p_0 + m}\left(\begin{array}{c} 1 \\ 0 \end{array}\right) \\ -\sqrt{p_0 - m}\left(\begin{array}{c} \cos\theta \\ \sin\theta\exp(i\varphi) \end{array}\right) \end{array}\right).$$

Let me remark, that $j = 1/2$ is minimum angular momentum for particle with $s = 1/2$. Now, one can easily calculate the average contribution of the spin operator to the total angular momentum:

$$\Sigma_3 = \frac{1}{2}\left(\begin{array}{cc} \sigma_3 & \cdot \\ \cdot & \sigma_3 \end{array}\right) \Rightarrow$$

4

$$\psi_{jlm}^{\dagger}\left(\mathbf{p}\right)\Sigma_3\psi_{jlm}\left(\mathbf{p}\right) = \frac{1}{16\pi p_0}\left[(p_0 + m) + (p_0 - m)\left(\cos^2\theta - \sin^2\theta\right)\right]$$

If a_p is the probability amplitude of the state ψ_{jlm}, then

$$\langle\Sigma_3\rangle = \int a_p^{\star}a_p\psi_{jlm}^{\dagger}\left(\mathbf{p}\right)\Sigma_3\psi_{jlm}\left(\mathbf{p}\right)d^3p = \frac{1}{2}\int a_p^{\star}a_p\left(\frac{1}{3} + \frac{2m}{3p_0}\right)p^2dp, \tag{6}$$

which means, that:

i) For the fermion at rest $(p_0 = m)$ we have $j = s = 1/2$, which is quite comprehensible, since without kinetic energy no orbital momentum can be generated.

ii) For the state in which $p_0 \geq m$, we have in general:

$$\frac{1}{3} \leq \frac{\langle s\rangle}{j} \leq 1.$$

where left limit is valid for the energetic (or massless) fermion, $p_0 \gg m$. In other words, in the states ψ_{jlm} with $p_0 > m$ part of the total angular momentum $j = 1/2$ is *necessarily* created by orbital momentum. This is a simple consequence of quantum mechanics.

Now, one can compare integrals (5) and (6). Since both integrals involve the same kinematic term, the interpretation of dependence on ratio m/p_0 in (6) is valid also for (5). Otherwise, the comparison is a rigorous illustration of the statement, that Γ_1 measures contributions from quark spins (and not their total angular momenta).

V. SUMMARY

I have studied spin functions in the system of quasifree fermions having total spin $J = 1/2$ - representing a covariant version of naive QPM. The main results are:

1) Spin functions g_1 and g_2 depend on intrinsic motion. In particular, the momenta Γ_1 corresponding to the static (massive) fermions and massless fermions, can differ significantly: $\Gamma_1(m \ll p_0)/\Gamma_1(p_0 \approx m) = 1/3$. It is due to splitting of angular momentum into spin and orbital part, as soon as intrinsic motion is present.

2) g_1 and g_2 are connected by a simple transformation, which is for $m \to 0$ identical to Wanzura - Wilczek relation for twist-2 term of the g_2 approximation. Relations for the $n-th$ momenta of the structure functions have been obtained, their particular cases are identical to known sum rules: Wanzura - Wilczek ($n = 2, 4, 6...$), Efremov - Leader - Teryaev ($n = 1$) and Burkhardt - Cottingham ($n = 0$).

3) Model has been applied to the proton spin structure, assuming proton spin is generated only by spins and orbital momenta of the valence quarks with $SU(6)$ symmetry and for quark effective mass $m \to 0$. Using a known parameterization of the valence terms as the input, the functions g_1, g_2 and the transversity δq are obtained.

[1] P. Zavada, Phys. Rev. D **65**, 054040 (2002).
[2] P. Zavada, Phys. Rev. D **67**, 014019 (2003).
[3] A.V. Efremov, O.V. Teryaev and P. Zavada, Phys. Rev. D **70**, 054018 (2004).
[4] S. Wanzura and W. Wilczek, Phys. Lett. B **72**, 195 (1977).
[5] H. Burkhardt, W.N. Cottingham, Ann. Phys. **56**, 453 (1970).
[6] A.V. Efremov, O.V. Teryaev, E. Leader, Phys. Rev. D **55**, 4307 (1997).
[7] A. Airapetian et al., Phys. Rev. Lett. **92**, 012005 (2004).
[8] A.V. Efremov, K. Goeke, P. Schweitzer, Eur. Phys. J. C **35**, 207 (2004).
[9] PAX Collaboration, Antiproton-Proton Scattering Experiments with Polarization, Letter of Intent (Darmstadt, 2004), available at http://www.fz-juelich.de/ikp/pax.
[10] P. Zavada, Phys. Rev. D **55**, 4290 (1997).
[11] L.D. Landau, E.M. Lishitz et al., Quantum Electrodynamics (Course of Theoretical Physics, vol. 4), Elsevier Science Ltd., 1982.